# Simplified dynamic analysis of high-speed railway bridges considering bending and torsional effects

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**Abstract.** Railway bridges are subjected to significant dynamic effects due to the passage of trains at high speeds (>250 km/h), which can induce resonance. Additionally, the presence of eccentric moving loads, skewed supports, and variations in cross-sectional geometry induces torsion in the bridges, coupled with bending. To understand the influence of all these effects on the Spanish railway bridges, here three common cross-sections are analyzed: slab on I-beams, slab on U-beams and single cell box girders, all designed for double track with a width of 14 m. The analysis of bending and torsion coupling is carried out by comparing 3D finite element models with semi-analytical models. The semi-analytical models are developed in the in-house software Caldintav [1]. On the other hand, the finite element 3D models are implemented in commercial software [2]. An important part of the analysis is to determine the torsional constant *J* for each section. In the case of box girder and slab on U-beams, the Bredt's equation is used, and in the case of slab on I-beams the *J* is calibrated with a finite element model. As a conclusion, it is specified that a correct definition of *J* is critical to have accurately results when considering semi-analytical models, especially when bending is coupled with torsion.

**Keywords:** Railway bridges, high-speed trains, bending, torsion, semi-analytical models, open and close cross-sections, Finite Elements

# 1 Introduction

The development of high-speed railway lines in Spain began in April 1992 with the inauguration of the Madrid-Seville line. Since then, the network has grown significantly, with a total of 3974 km. This efficient and successful mode of transport set new goals, such as achieving interoperability within the European Union, a target that was established in 1996 [3].

This study focuses on analyzing representative bridges within the Spanish railway network, characterized by three typical cross-sectional profiles, to capture both bending and torsional effects. The bridges examined were specifically located along the Madrid-Barcelona high-speed line.

The dynamic analysis of these bridges employs two methodologies: a semi-analytical model and a numerical 3D Finite Element Model (FEM). The semi-analytical method employs transient modal analysis, based on beam assumptions which incorporate the geometric, mechanical, and dynamic properties of the structure. It offers significant computational efficiency, both for defining the model as for the time-step solution algorithm. However, the beam model assumptions may underestimate some structural 3D features. In contrast, FEM provides a more detailed and accurate representation, but at a significantly higher computational cost.

The main objective of the study is to obtain a semi-analytical model incorporating the effects of bending and torsion to capture the real behavior of a bridge, for which the focus is on incorporating torsion into the model by including the torsional constant $J$ using the known equations (Box girders and Slab on U-beams) or by calibrating using the FEM (Slab on I-beams). The results of this study aim to contribute to the improvement of design and analysis methods for high-speed rail infrastructure, to ensure the safety and cost-effectiveness of future projects.

# 2 Structural typologies

The evaluation of the dynamic behavior of bridges on the Madrid-Barcelona high-speed railway lines is essential to ensure the safety and efficiency of rail transport. This line was characterized by a variety of structural typologies, including simply supported bridges and continuous bridges. For the present study, simply supported bridges with moderate spans (10 to 30 m) from the line were selected, due to their higher susceptibility to resonance issues [4][5]. The characterization of the cross-sections reveals three predominant typologies in the Spanish railway network: slab on I-beams, slab on U-beams and box girder (Fig.1), all of which were constructed with reinforced concrete. Three simply supported bridges (doubled track) have been selected as representative of each of these categories, and their general characteristics are included in Table 1 and Table 2. Just the slab on U-beams and the box girder typology had diaphragms as lateral bracing.

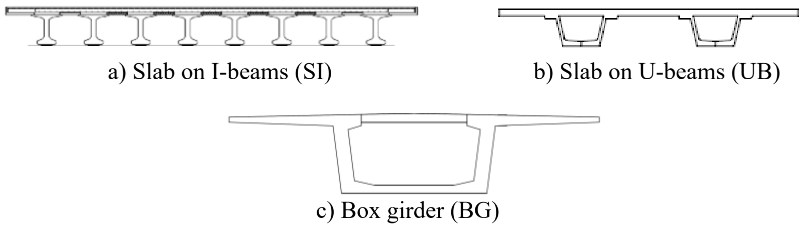



**Fig. 1.** Cross-Sections of the analyzed bridges, Madrid–Barcelona ADIF Line [6].

**Table 1.** Detailed information of the selected bridges.

| ID | Viaduct name | no. of spans | Viaduct length (m) | Spans | |
|---|---|---|---|---|---|
| | | | | Min (m) | Max (m) |
| SI | Santa Catalina Viaduct | 47 | 512,55 | 6,48 | 15,30 |
| UB | Paso inferior Nº 4 | 2 | 42,79 | 20,38 | 20,38 |
| BG | Jalón Viaduct | 64 | 2235,00 | 28,63 | 48,00 |

# 3 Semi-analytical model

## 3.1 Model

In the dynamic analysis of beams, the correlation between modal parameters (natural frequencies, damping, and mode shapes) and the geometric and material properties are established through the solution of the differential equation of motion, as shown below. The differential equations are solved separately because the bending and torsional modes are uncoupled.

$$m_l \frac{\partial^2 y(x,t)}{\partial t^2} + c_y \frac{\partial y(x,t)}{\partial t} + \frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2 y(x,t)}{\partial x^2}\right) = F(x,t) \tag{1}$$

$$m_l r^2 \frac{\partial^2 \theta(x,t)}{\partial t^2} + c_\theta \frac{\partial \theta(x,t)}{\partial t} - \frac{\partial^2}{\partial x^2}\left(GJ \frac{\partial \theta(x,t)}{\partial t}\right) = M(x,t) \tag{2}$$

Where $y(x,t)\ and\ \theta(x,t)$ are vertical deflection and torsional rotation of the bridge, $c_y, c_\theta$ are damping coefficients and $F(x,t), M(x,t)$ are vertical and torsional loads.

For a simply supported beam under free vibration, its mode shapes could be described by the Eq. (3), which satisfies the boundary conditions imposed by its supports.

$$\emptyset^i(x) = \sin\left(\frac{i\pi x}{L}\right) \quad (3)$$

The natural bending frequencies for free vibration are

$$f_b^i = \frac{i^2\pi}{2L^2}\sqrt{\frac{EI}{m_l}} \quad (4)$$

In an analogous manner, the natural torsional frequencies are

$$f_{tb}^i = \frac{i}{2L}\sqrt{\frac{GJ}{m_l r^2}} \quad (5)$$

A detailed description of the variables is provided in Table 2.

In this method, the train is represented as a sequence of moving loads traveling across the bridge with constant speed, exciting the bridge's natural vibration modes. When the frequency of the passing train approaches the natural frequencies of the bridge, the resonance is generated. The dynamic response of the bridge is obtained as the superposition of the modal responses at each time instant [7].

The implementation of a semi-analytical model enables a rapid estimation of the structural response of a bridge subjected to bending and torsional actions. In this approach, the bridge is represented as a simply supported three-dimensional beam, accounting for three bending modes and one torsional mode. The semi-analytical model was developed using a Python-based open-source code called Caldintav [1]. The parameters required for the analysis include the geometric and mechanical properties of the bridge cross-section, as indicated in Table 2. For each analysis it uses three bending modes and one torsion mode.

**Table 2.** Geometric and mechanical parameters considered for the bridges.

| Parameters | Values | | |
|---|---|---|---|
| | SI | UB | BG |
| Span length *L* (m) | 15,30 | 20,38 | 28,63 |
| Mass p.u. length $m_l$ (kg/m) | 28042,00 | 28202,75 | 35302,75 |
| Young's Modulus *E* (GPa) | 31,3 | 32,3 | 31,7 |
| Shear Modulus *G* (GPa) | 12,0 | 12,9 | 12,2 |
| Inertia *I* ($m^4$) | 0,94 | 1,64 | 10,51 |
| Damping $\zeta$ (%) | 1,33 | 1,00 | 1,00 |
| Eccentricity *e* (m) | 2,35 | 2,35 | 2,35 |
| Skew angle (º) | 0 | 42,5 | 0 |
| Overhang (m) | 0,35 | 0,69 | 1,00 |
| Torsional radius of gyr. *r* (m) | 3,91 | 3,85 | 3,34 |
| Torsional stiffness const. *J* ($m^4$) | 1,402 | 1,89 | 18,89 |

### 3.2 Torsional constant *J*

The incorporation of the torsional effect in the semi-analytical model is achieved through the modal coupling of natural frequencies. This study focused on determining the torsional frequency based on relevant geometric properties, such as the radius of gyration *r* and the torsional constant *J*.

In this context, the parameter of interest is the *J,* which influenced the dynamic behavior of the system. For the box girder and slab on U-beams, they were analyzed as closed sections Bredt [8]:

$$J = \frac{4A_m^2}{\sum \frac{L}{t}} \tag{6}$$

Where $A_m$ is the area of the enclosed cross-section, $L$ is length of each wall segment and $t$ is the thickness of the corresponding wall segment

In the case of the slab on I-beams, it was originally proposed to consider them as open sections. Therefore, each beam and the slab contributed a torsional constant *J,* and the summation of these values provides an approximation for the calculation [8].

$$J = \frac{1}{3} \sum L_i {t_i}^3 \tag{7}$$

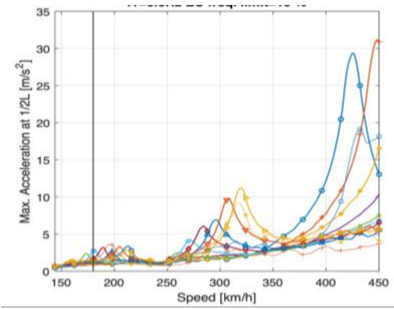
However, the obtained value proves to be inconsistent with the finite element model, then *J* is calibrated based on the latter.

### 3.3 Trains

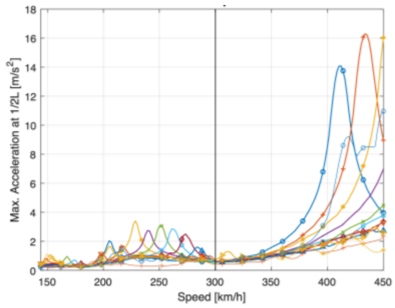
The analysis considers the passage of the ten HSLM train types as well as the real trains S100R (A100), S103 (A103), and S112 (A112). Each bridge is evaluated for 13 trains with a speed range from 144 to 450 km/h, with an increment of 1.8 km/h. This means that each bridge was tested a total of 2223 times.

### 3.4 Results

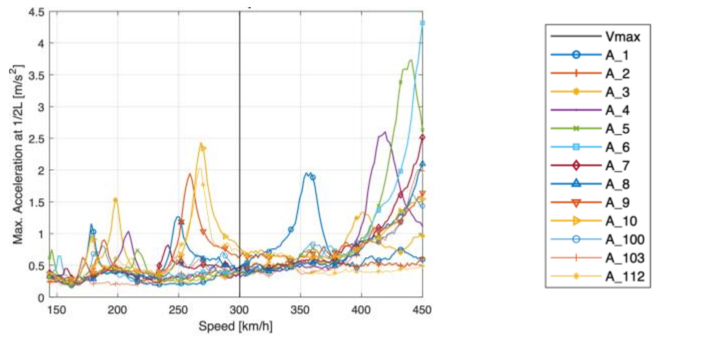
The envelope of maximum accelerations at the rail centerline is shown in Fig. 2. The maximum accelerations coincide reasonably with the resonant speeds calculated according to Eq. 6.9 of Eurocode EN 1991-2 2003 [9]. Subsequently, the train that induces the highest acceleration is selected for a more detailed analysis using a 3D finite element model. The maximum speed ($V_{max}$) in each case is represented by the vertical line in the figure. An important consideration in semi-analytical models is the calculation time. In the case of the slab on I-beams, slab on U-beams and Box Girder, the analysis time were 2,55; 2,85; and 2,92 minutes, respectively. All calculations were performed on a system equipped with an Intel Xeon E5-2643 v4 processor (3.4GHz base frequency, 6 cores/12 threads per CPU).




a) Slab on I-beams (SI).




b) Slab on U-beams (UB).




c) Box girder (BG).

**Fig 2.** Envelopes of Maximum Accelerations with the semi analytical model.

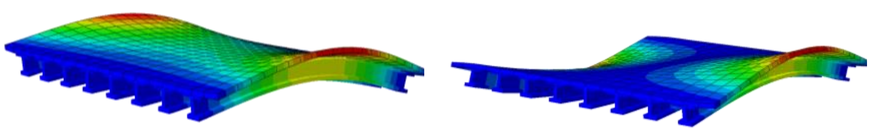
# 4 3D Finite Element Modelling (FEM)

## 4.1 Model

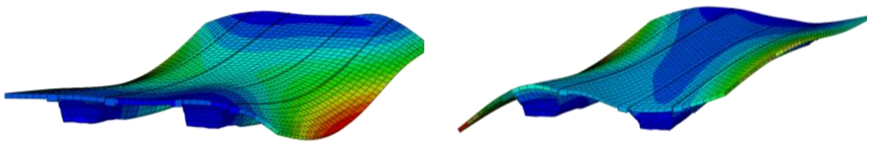
The 3D finite element models have been developed using shell elements and beam elements in Abaqus software [2], see Table 2 for the geometrical and mechanical properties. The support conditions are assigned according to the characteristics and locations specified in the design drawings [6]. Diaphragms are incorporated where required for the slab on I-beams and for the slab on U-beams. For this last, the corresponding skew angle is also considered. In all structures (SI, UB, and BG), S4R finite elements (4-node quadrilateral shell elements with reduced integration) have been used to model the slabs and beams, except in the SI case where the beams have been modeled using B31 elements based on Timoshenko beam theory. Diaphragms in the UB model have been represented using C3D8R solid elements (8-node hexahedral elements with reduced integration). A structured mesh has been employed to ensure high-quality analysis. The main beams have been defined as prestressed concrete, while the slabs have been modeled with conventional concrete. Dynamic moving loads have been applied with a Python script -developed at the Computational Mechanics Group of the UPM-, and a numerical modal analysis has been conducted to identify vibration modes and evaluate the dynamic behavior in the simulated environment, considering a velocity range of 144 km/h to 450 km/h, with an increment of 1.8 km/h.

### 4.2 Results

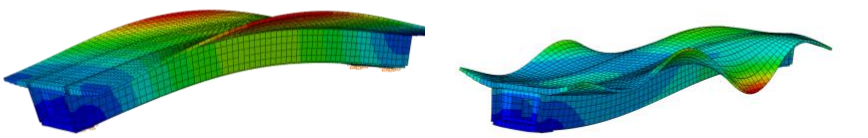
The results of the bending and torsional mode shapes and frequency values can be observed in the following figures: slab on I-beams (Fig. 3a), slab on U-beams (Fig. 3b), and box girder (Fig. 3c). The envelope of the maximum accelerations is presented in the following section (Fig. 4). The calculation times for FEM simulations using an Intel Xeon E5-2643 v4 processor (3.4GHz, 6 cores/12 threads per CPU) were: Slab on I-beams: 108,5 min; Slab on U-beams: 245,10 min; Box girder: 570,00 min.



a) Slab on I-beams: bending 6,58 Hz, torsion: 6,47 Hz



b) Slab on on U-beams: bending 6,35 Hz, torsion: 5,93 Hz



c) Box girder: bending 5,34 Hz, torsion: 16,19, Hz

**Fig 3.** Modal Shape of the different sections: 1st bending mode and 1st torsion mode.

## 5 Discussion of results

The frequency results are presented in Table 3. A comparison of the acceleration envelopes between the semi analytical model and the FE model results are also discussed here. The fundamental bending and torsional frequencies calculated with Caldintav, and the frequencies obtained with Abaqus that closely approximate these modes are indicated, considering that in Abaqus neither pure bending nor torsional modes are obtained due to the modal coupling inherent in its analysis.

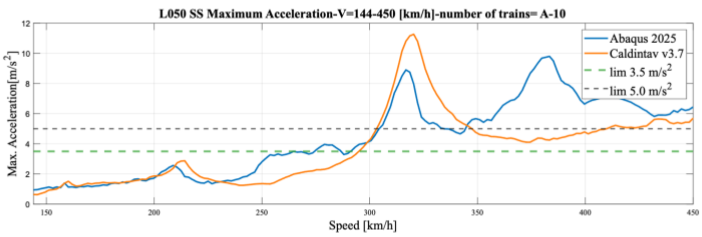
**Table 3.** Bending and torsional modes: Caldintav and Abaqus.

| Model | Mode | Caldintav (Hz) | Abaqus (Hz) | Difference |
|---|---|---|---|---|
| SI | Bending 1 | 6,579 | 6,482 | 1,49 % |
| | Torsion 1 | 6,470 * | 6,470 | 0* % |
| UB | Bending 1 | 6,35 | 6,29 | 0.95 % |
| | Torsion 1 | 5,93 | 5,79 | 2.41 % |
| BG | Bending 1 | 5,514 | 5,347 | 3.123% |
| | Torsion 1 | 13,360 | 16,191 | 17.486% |

*Calibrated with FEM

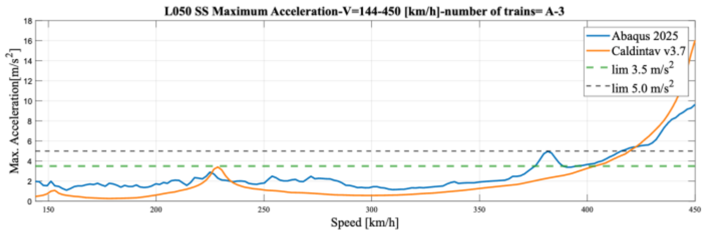
The modal frequency's results yield acceptable values, given that the FEM provides a more detailed representation of the model. For the slab on I-beam, as previously mentioned, the torsional frequency is incorporated by calibrating the *J* parameter based on the FEM model. In all other cases, the equation proposed by Bredt is used. The vibration modes allow for an initial approximation of the dynamic behavior of both models.

The important result is the derivation of maximum acceleration envelopes, given the 5 m/s² service limit for existing ballasted tracks [10]. For each bridge, comparative analyses are performed against respective train configurations as illustrated in Fig. 4. The comparison between Abaqus and Caldintav within the velocity range of 144 to 450 km/h reveals peak acceleration values calculated by Caldintav that vary significantly depending on the structural typology. For the SI (Fig. 4a), a maximum value of 11,27 m/s at 320,40 km/h. For the UB (Fig. 4b), a maximum value of 16,040 m/s² at 450 km/h. Finally, for the BG (Fig. 4c), a maximum value of 2,436 m/s² at 268,20 km/h. These results indicate that the Caldintav model, by incorporating bending and torsional modes, tends to provide a conservative estimation of the maximum accelerations induced by high-speed railway traffic. However, it is important to note that despite extending the analysis up to 450 km/h, the service speed limits for trains on these structures are 180 km/h for SI and 300 km/h for the UB and BG, respectively.

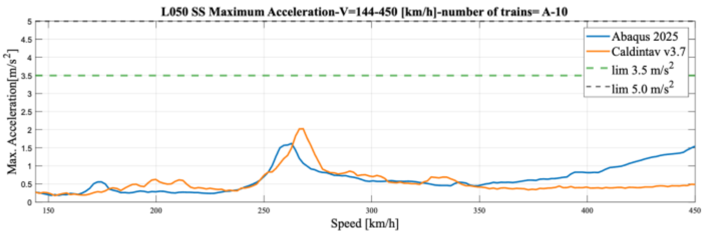



a) Santa Catalina Viaduct (SI)-train: HSLMA-10 -$V_{max}$ =180 km/h

**Fig 4.** Comparison of acceleration envelopes between the FEM and the semi analytical model: a) Santa Ana Viaduct, b) Puente Inferior Nº 4 and c) Jalon Viaduct.




b) Paso inferior Nº 4 (UB)-train: HSLMA-3 - $V_{max}$. =300 km/h




c) Jalón Viaduct (BG)-train: HSLMA-10- $V_{max}$ =300 km/h

**Fig 4 (continuation).** Comparison of acceleration envelopes between the FEM and the semi analytical model: a) Santa Ana Viaduct, b) Puente Inferior Nº 4 and c) Jalon Viaduct.

## 6 Conclusion

In this work, three models related to the cross-section of the bridges are analyzed: a slab on I-beams, a slab on U-beams, and a box. The modal shapes and frequencies results show that bending and torsional frequencies are quite similar between the semi-analytical and FEM models for the SI and UB. In contrast, the BG model exhibits a slightly higher torsional frequency in the FEM model, attributed to the additional stiffness provided by the flanges of the section.

In terms of calculation efficiency, the semi-analytical 3D beam models have a clear advantage, as they require significantly less calculation time than FEM (Slab on I-beams FEM:108,50 min Caldintav: 2,55 min). This calculation time makes it possible to analyze large train sets at different speed levels, resulting in a smoother graphical representation and improved operational efficiency.

The calculation of torsional stiffness $J$ for slab sections on SI is not straightforward in semi-analytical beam models for open cross-sections, and this issue remains as a further study. However, by calibrating with finite element models, it is possible to optimize the results and conduct a detailed analysis of a set of bridges using the semi-analytical model. This demonstrates the applicability and effectiveness of the model in structural studies.

The integration of additional parameters, such as torsion and skew angle of supports, enhances the precision of semi-analytical models, allowing for a more refined representation of structural behavior under dynamic loading conditions. These parameters are essential for obtaining results that more accurately reflect realistic conditions and for modeling effects that may not be captured by simpler models.

In summary, semi-analytical 3D models are a valuable tool for the structural analysis of bridges, characterized by their computational efficiency, their increased accuracy due to the inclusion of additional parameters and their ability to be calibrated with finite element models to obtain more precise results.

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